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Market-Making and Decentralized Trade

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MARKET–MAKING AND DECENTRALIZED TRADE *

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Abstract:
A model of a decentralized economy is presented. The basic features of decentralized trade taken into account are, that agents have a limited knowledge of their economic environment, that such knowledge requires some kind of communication or interaction between agents, and that agents face uncertainty as to their immediate trading opportunities. The paper stresses the importance of an explicit consideration of the communication and trading structure in models of decentralized trade. We consider trade in a homogeneous commodity. Besides deciding upon their effective demands, agents may create their own markets by spreading information about themselves. The market structure in a Symmetric Nash Equilibrium is characterized as follows. The economy splits up in a number of possibly overlapping, imperfectly competitive markets. These markets are all made by sellers, as buyers won't engage in such activity. Both suppliers and demanders may be rationed at the same time, i.e. markets are not orderly.

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1. Introduction

One of the main tasks of economic theory is to explain the outcomes of a decentralized economy. The best developed models which address this problem are the Walrasian general equilibrium models. These models start with a set of separate individuals, each characterized by preferences, technologies and endowments (the so-called *primitives* of the economy). To explain both the actions of the individuals and the outcomes of these actions resort is taken to a number of concepts and rules which imply a very specific trading and communication structure. One of the distinguishing characteristics of this is that all trade and communication take place centrally through *The Market*, which is a public good kindly provided by the auctioneer. This structure may well be represented by a graph in which nodes denote individual agents and arcs communication between these agents (see e.g. Kirman [1983]).

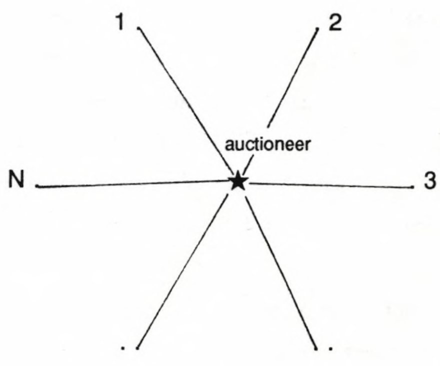

fig. 1.1 Walrasian structure

With respect to information the Walrasian trading structure is very efficient.[1] There are only information signals needed directed to or from the auctioneer, and the only information the auctioneer needs in order to evaluate the incoming signals (individual demands) and to calculate outgoing signals (prices) is the aggregate demand on each market.[2]

This efficiency is probably the main reason of attraction to the Walrasian structure. Unfortunately, this structure is at the same time also highly unrealistic and restrictive.

Without the auctioneer and the communication lines connecting the auctioneer with the individual agents, the auctioneer's functions are no longer fulfilled. There is no tâtonnement process, there is nobody to announce relevant information centrally, and nobody is taking care of an orderly, frictionless clearing of the markets. Trade is no longer centralized and individual agents have to take care themselves of their trades. In general, minimum requirements for trade being feasible are that, first, two or more agents meet of which one is a seller and another a buyer, and second, they agree upon the rate of exchange and the quantities to exchange. Therefore, to start with, one has to specify how agents enter in contact with each other. That is, having lost the Walrasian 'star', one needs an alternative story of how markets are organized in a decentralized economy (see Fisher [1989] or Gould [1980]).

We will study this problem for a decentralized economy which is characterized by the following basic properties:

(i) There is a large number of agents and a large number of different commodities, each agent being characterized by preferences, technologies and endowments.

(ii) Each agent is interested in only a small number of commodities, while the fraction of agents interested in a commodity is small for each commodity (cf. Fisher [1983]).

(iii) The economy is not a priori organized by an auctioneer, intermediary, specialized trader, central distributor, or anonymous random matching mechanism (e.g. Gale [1985]), but instead the economy to consider is one with decentralized trade which depends upon the decisions of individual agents who act on a strictly do-it-yourself basis.

(iv) Agents, although knowing about the existence of other agents, have no pre-communication knowledge of each other's individual characteristics like e.g. effective demands (let alone endowments and preferences).

(v) As long as an agent doesn't have any information about the characteristics of any other agent, he is not able to find a trading partner.

(vi) Individual agents may communicate with each other.

(vii) Communication is costly.

Thus, just like in the Walrasian models, we start with a set of individual agents with given characteristics. After all, that is what economic theory is about: rational and self-seeking individual agents (see Hahn [1983]). Basically, the assumed properties of a decentralized economy imply that there is an information problem and that this problem has to be solved, in one way or another, by the individual agents in order for trade to be possible. All trade involves transaction costs. These transaction costs are the costs of sending or gathering information about potential trading possibilities. Without such information no trade is possible. A key characteristic is *(v)*. That characteristic may be explained by reasoning that expected costs of uninformed search are greater than its expected gains. This may be due to the fact that the 'psychological' costs (disutility) of accosting a randomly chosen agent to bother him with a question like "Could you please sell me a refrigerator?" are high, while the probability that such an agent will indeed be interested in such a transaction is low. Although this is related to the other characteristics, it is convenient to assume it directly.[3] Thus, by communication with other agents an individual agent creates the possibility to meet potential trading partners. In general, when there are possibilities to trade in a certain commodity, it is said that *a market* for that commodity exists. Hence, by establishing communication with other agents, individual agents create in a certain sense their own markets. In this view a market is not a central place where a certain good is exchanged, nor is it simply the aggregate supply and demand of a good. A market is constituted by communication between individual agents. As Blin [1980] puts it:

«Markets rarely emerge in a vacuum, and potential traders soon discover that they may spend more time, energy, and other resources discovering or "making" a market than on the trade itself. This predicament is shared equally by currency traders, do-it-yourself realtors, and streetwalkers! Their dilemma, however, seems to have gone largely unnoticed by economists, who simply assume that somehow traders will eventually be apprised of each other's existence - to their mutual benefit or subsequent regret.» (p. S193)

Characteristic *(iv)* implies that an individual agent is not able to direct his communication towards a subset of agents with some specific characteristics. Hence, although an individual agent may very well choose the agents with whom to communicate in a deterministic manner, we may consider the choice of agents with whom to communicate to be stochastic. Thus, we can represent the communication structure like in figure 1.2, where each dotted line denotes a communication *possibility*.[4]

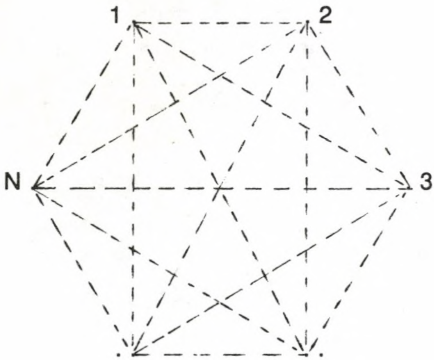

fig. 1.2 non-Walrasian structure

In the sketched framework individual agents will face market uncertainty, i.e. they will be uncertain about their immediate trading possibilities. The presence of market uncertainty has two causes. First, the tâtonnement process, which precluded market uncertainty as the auctioneer signaled the individually relevant information to each agent before trade might take place, has been abolished. Second, the contents of the relevant, i.e. individually needed, information has changed due to the changed structure of trade. Individually relevant information would be complete when each

agent knew the current price vector, the complete vector of effective demands of all other agents, and where and when to meet these agents.

This market uncertainty is endogenous uncertainty in two senses. First, it is inherent in the structure of the trading process itself. In this respect markets cannot be complete, as additional markets for insurance or hedge transactions would face the same kind of market uncertainty problems (see Foley & Hellwig [1975] or Dehez [1980]). Secondly, the level of market uncertainty is in a certain sense endogenous. The agents' state of information about their transaction possibilities depends upon the amount of communication in the economy. All communication is between individual agents, and its amount just depends upon how much information costs these individual agents decide to incur.

To stress the importance of the presence of market uncertainty further, the basic framework will also have the following characteristics.
(viii) There is no exogenous uncertainty, i.e. there is no 'state-of-nature' uncertainty. In this respect markets are complete.
(ix) Individual agents know the aggregate state of the economy (e.g. aggregate demands, total numbers of sellers/buyers).
(x) All commodities are known by all agents (cf. Gary-Bobo & Lesne [1988]).
(xi) There is no quality uncertainty (cf. Spence [1974]).
(xii) The transaction costs mentioned are information costs and there isn't any kind of real transaction costs (see Shubik [1975]).

A methodological account of the choice of the assumed characteristics of the basic framework may be useful. Contrary to some other contributions in a non-Walrasian perspective, the key to the selection of assumptions is not to find those which will guarantee that certain 'Walrasian' efficiency properties will hold.[5] Instead, the idea is to focus on usefulness of assumptions in order to understand and analyze the apparent presence of information and coordination problems in decentralized economies. This might be considered as part of what could be called a program of "*recoverability*" (Varian [1984]), i.e. the search for a list of restrictive assumptions with which certain observed phenomena could be explained as the result of optimizing behavior.

In section 2 we will make these general assumptions concrete and specify more precisely how individual agents may create their markets. In section 3 we will derive an equilibrium market structure from the optimizing behavior of individual agents, and analyze its characteristics. Section 4 concludes.

2. The Model

2.1 Agents and Commodities

Assuming time to be divided into an infinite sequence of discrete periods indexed τ, $\tau \in \{1, 2, ...\}$, we consider an economy with a single homogeneous, perishable commodity in one period. A set A of N agents, each characterized by preferences, technologies and endowments, is divided into two disjoint classes (see e.g. Gale [1985]): a set B of m firms and a set D of N–m consumers. Thus, $|B| = m$, $|D| = N-m$, $B \cap D = \varnothing$, $B \cup D = A$ and $|A| = N$. We assume that N is large.

Given the agents' preferences, technologies and endowments, any given consumer i can be characterized by a threshold price $\overline{p}_i$. This threshold price $\overline{p}_i$ corresponds to the utility U_i this agent would derive from his consumption of one unit of the commodity, and is the price above which this agent would certainly not purchase a unit of the commodity (see e.g. Gale [1985] or Kormendi [1979]). Formally $\overline{p}_i$ is defined by:

$$U_i(0, \omega_i) = U_i(1, \omega_i - \overline{p}_i) , \qquad [2.1]$$

where the first argument of the utility function concerns the commodity of our consideration and the second represents a basket of other goods or 'income'. We assume that the threshold price is 0 for all consumers with respect to any further unit of the commodity. Formally:

$$U_i(1, \omega_i - \overline{p}_i) = U_i(a, \omega_i - \overline{p}_i) \;\forall\; a \geq 1 \;\forall\; i \qquad [2.2]$$

Thus, the number n of interested consumers depends upon the price p, and aggregate demand may be written n(p), which we assume to be objectively known by all sellers. In other words, given the characteristics of the individual agents, for each price p the set D of consumers consists of n(p) potential buyers and N−m−n(p) agents not interested at all in the commodity.

The m firms produce and sell the commodity. They are assumed to be identical in that they face the same technology. The cost C of producing z units of output is given by the function C(z), where z is a discrete variable ($z \geq 0$). We assume $C(0) \geq 0$ and $\Delta C(z)/\Delta z > 0$ for all $z \geq 0$, for all $\Delta z > 0$. Production decided upon at the beginning of the period is immediately available for sale, while unsold stocks perish at the end.

In order for trade to take place, firms and consumers must meet, and they must agree upon the terms of trade. We assume that the price p of the commodity is given and equal for all agents, and that it is known to all agents. This assumption is made in order to stress the logical distinction between trading uncertainty and price uncertainty. We focus upon trading uncertainty, which means being uncertain as to whether or not an agent will be able to trade what he wants at the going price (see Hahn [1980]).

Agents know that other agents exist, but they do not know any of the characteristics of these agents. In particular, they do not know which agent belongs to which class. Meeting potential trading partners, i.e. agents of the right class, is unfeasible if no agent has any information in this respect. Thus, we have to specify the way in which agents communicate with each other, by which they create their own markets.

2.2 Communication

Suppose for the moment that only the sellers may send information. This seems without doubt restrictive, although it conforms to what we usually observe in reality, but in section 3.4 we will show that it is not. Each seller may send information signals to some other agents at the beginning of the period, each signal being directed to one agent. A signal contains, first, the 'name and address' of the sending agent, and secondly, the fact that he belongs to the class of firms B. Thus, signals reveal the type of a given agent, like, for example "*Mr. A, 22 Oxfordstreet sells refrigerators*".[6]

The rationale for signaling should be clear: the signals are the only means of direct communication between individual agents. Agents who perceive no signals and are sending no signals can't find a trading partner.

We assume that signaling is costly, the cost being an increasing function of the amount of signals sent. The cost K of sending s signals is given by the function $K(s) = k \cdot s$, where s is a discrete variable ($s \geq 0$) and $k > 0$. Thus, about the signaling technology we assume that it always costs k to send one additional signal, while K(0) = 0. Receiving signals, on the other hand, is costless.

2.3 Trade

Agents make their decisions concerning communication and effective demand at the beginning of each period (see next section). During each period they try to buy or sell in their markets. We assume that the trading possibilities for each agent are dependent only upon the communication and demands in the given period. Thus, sellers have no reputation and there are no customer relations. Moreover, demand above the firm's available supply is simply forgone and cannot be backlogged.

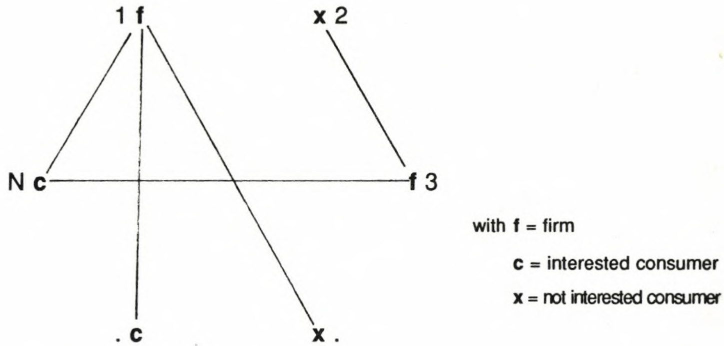

fig. 2.1 markets in non-Walrasian structure

When the firms have sent their signals the economy might look like figure 2.1. In this example firm 3's market comprises consumers 2 and N, while consumer N's market consists of firms 1 and 3, etc.

We assume that each consumer who has received one or more signals may visit one firm ('shopping'). The order in which buyers make their visit is random.[7] When a firm is sold out customers will turn home unsatisfied, otherwise they may buy one unit. Thus, trade in our model is bilateral.[8]

3. Strategies and Equilibrium

3.1 Objectives and Strategies

A consumer's utility would increase in the current period if he could buy one unit of the commodity at a price below his threshold price $\bar{p}$. If a consumer receives one or more signals at the beginning of the period, and if the price is indeed below his threshold price $\bar{p}$, then he randomly chooses among these signals a firm to visit.[9] If this firm is sold out the consumer will be left unsatisfied in this period, otherwise he will buy one unit.[10]

Firm i's objective is to maximize its expected current profit V_i , which is equal to its expected gross revenue R_i minus its production cost $C(z_i)$ minus its market-making cost $K(s_i)$, by deciding upon its effective supply z_i and signaling s_i. Moreover, it has to decide to which agents it sends these signals. We assume that the destination of each signal is chosen at random.[11] Thus, a strategy t_i of firm i is a pair (z_i, s_i). The costs are dependent only upon firm i's own strategy, but the revenue of its market-making and production activity, i.e. firm i's actual sales ($p \cdot x_i$, to be defined later), are a function of the vector **t** of the strategies of all firms (including firm i itself). (Vectors are denoted by **bold-face** letters. An overview of the notation can be found in appendix E.) Thus, firm i's objective function is:

$$V_i(\mathbf{t}) = R_i(\mathbf{t}) - C(z_i) - K(s_i) \,, \qquad [3.1]$$

$$\text{where } R_i(\mathbf{t}) = p \cdot x_i(\mathbf{t})$$

We now give two definitions and present the objective of this part of the paper.

Definition 3.1: A Nash Equilibrium (NE) is a vector of strategies $\mathbf{t}^* \equiv (t_1^*, ..., t_i^*, ..., t_m^*)$ such that, for each i, firm i maximizes its payoff V_i by choosing strategy t_i^* given the strategies of the other firms $\mathbf{t}_{-i} = \mathbf{t}_{-i}^*$, i.e. $V_i(t_i^*, \mathbf{t}_{-i}^*) \geq V_i(t_i, \mathbf{t}_{-i}^*)$ $\forall$ i $\forall$ t_i .

Definition 3.2: A Symmetric Nash Equilibrium (SNE) is a NE $\mathbf{t}^*$ such that $t_i^* = t_j^*$ $\forall$ i $\forall$ j.

We restrict our attention to the existence and characterization of a SNE. Given the assumed non-Walrasian trading structure and the assumptions about the primitives of the economy, we will show for which parameter values such a SNE exists.

The question to answer concerning existence of a SNE is: Does a strategy t^* exist such that if all other firms choose t^* it is optimal for firm i also to choose t^* ? In order to consider the optimization problem of firm i in section 3.3, we first make explicit how firm i's trading opportunities, and hence its payoff V_i , depend upon the vector of strategies **t**.

3.2 Stochastic Trading Opportunities

Firm i's expected gross revenue is equal to its expected sales: $R_i(\mathbf{t}) = p \cdot x_i(\mathbf{t})$. Given the assumptions made in the previous sections about the nature of the commodity and the trading structure, it is clear that firm i can't sell more than is demanded by its customers $(q_i(\mathbf{s}))$ or than it has produced at the beginning of the period (z_i). That is, $x_i(\mathbf{t}) = \min\{q_i(\mathbf{s}), z_i\}$. Therefore, we have to specify the demand directed towards firm i $q_i(\mathbf{s})$.

Proposition 3.1: The demand directed towards firm i $q_i(\mathbf{s})$ is a random variable which may be represented by a Poisson distribution with parameter $\mu_i = \mu(s_i, S_{-i}) = (s_i/S) \cdot n(p) \cdot (1 - e^{-S/N})$, where S_{-i} denotes the aggregate number of signals sent by the other firms and $S = s_i + S_{-i}$.

Proof: The complete proof can be found in appendix A. Here we give a sketch. Firm i sends s_i signals at random into the population. Each signal may result in a consumer, who wants to buy one unit, visiting firm i or may remain without any reaction. This depends upon the probability that the receiver of such a signal is an interested consumer and the probability that he will choose the signal from firm i among the signals he receives. The latter, clearly, also depends upon the aggregate signaling activity of the other firms. It turns out that the probability that any given signal sent by firm i will lead to a consumer visiting firm i is: $\Pr(S) = (n(p)/S) \cdot (1 - e^{-S/N})$. Firm i sends s_i signals, and the number of buyers visiting firm i may be approximated by a Poisson distribution with parameter $\mu(s_i, S_{-i}) = s_i \cdot \Pr(S) = (s_i/S) \cdot n(p) \cdot (1 - e^{-S/N})$. □

This result can be interpreted straightforwardly. The potential aggregate demand in the economy, given the price p, is n.[12] The probability that any given potential consumer won't receive any signal at all, and thus will not find his way to a market is $e^{-S/N}$. Hence, aggregate market demand is $n \cdot (1 - e^{-S/N})$. Finally, each firm's expected market share turns out to be equal to his share in the aggregate market-making activity (s_i/S).

Note that the probability of any given signal from firm i having success is a function only of the aggregate number of signals S sent by all firms. Hence, $Pr_i(S) = Pr(S)$ for all i. This is because each interested consumer handles all his received signals identically, putting them all in an urn and drawing just one signal. Notice also that μ_i turns out to be a function only of the number of signals sent by firm i itself (s_i) and the aggregate number of signals sent by all other firms (S_{-i}). Thus, the vector of strategies chosen by the other firms $\mathbf{t}_{-i}$ enters firm i's decision problem only through the aggregate market-making signaling activity.[13]

The resulting transaction possibilities for any given agent are stochastic. Thus, agents are uncertain as to whether they will be able to trade as much as they want. There are, as we have seen, two direct causes for this uncertainty. First, communication is stochastic, i.e. signals are randomly distributed because agents don't know each other's characteristics. Second, given that an agent has found or established a market, he or his potential trading partners may have fulfilled already their demand before they happen to meet, i.e. shopping is a stochastic process.

The trading possibilities for firm i are derived explicitly from assumptions about the underlying communication and trading structure of the economy, instead of assuming directly a functional form of each agent's trading possibilities. The stochastic demand for firm i's output depends upon one of the (non-price) decision variables of the firm itself. This stochastic demand is not generated by sending an effective demand (i.e. supply) to the market, but by creating the market itself.[14]

As a result, firm i's gross revenue may be written: [3.2]

$R_i(z_i, s_i, S_{-i}) = p \cdot \{\Sigma_{q=0}^{z} q_i \cdot f[q_i| \mu(s_i, S_{-i})] + z_i \cdot \Sigma_{q=z}^{\infty} f[q_i| \mu(s_i, S_{-i})]\}$,

where $f[q|\mu]$ denotes the p.d.f. of q with parameter μ

Observe that the stochastic trading mechanism has an anonymity property built in. That is, agents who have the same effective demand and have sent out the same number of signals can expect the same realizations. This is due to the fact that trading possibilities only depend upon current period variables, that all signals are for each firm independently distributed, each agent being equally likely to receive such signals, that the firms to visit are chosen independently by all buyers, each firm being equally likely to be chosen among the firms in the buyer's market, and that the order in which buyers make their visits is determined randomly and does not depend upon the agents themselves.

We further characterize the stochastic demand directed to firm i through the following claims.

Claim 3.1 a: For given S_{-i} , $\mu(s_i, S_{-i})$ is a one-to-one function of s_i , with

$\mu(0, S_{-i}) = 0$,

$0 \leq \Delta\mu(s_i, S_{-i})/\Delta s_i \approx Pr(S) \leq 1$,

$\Delta^2\mu(s_i, S_{-i})/\Delta s_i^2 < 0$, and

$\lim_{s_i \to \infty} \mu(s_i, S_{-i}) = n$

b: For given s_i , $\mu(s_i, S_{-i})$ is a one-to-one function of S_{-i} , with

$\Delta\mu(s_i, S_{-i})/\Delta S_{-i} < 0$, and

$\lim_{S_{-i}\to\infty} \mu(s_i, S_{-i}) = 0$

Proof: See appendix A.

Thus, a firm which doesn't signal doesn't get any demand. The expected change in the demand directed to firm i as a result of sending one additional signal is positive but less than 1, and it depends only upon the aggregate signaling activity in the economy. Notice that for given S this is equal for all firms, and that it does not matter which firm sends how much signals, and in particular it doesn't matter how much of the S signals are sent by firm i itself. A firm may eventually capture the whole aggregate demand by signaling more and more, given the strategies of the other firms. However, the more other firms signal, the less will be firm i's expected demand.

Suppose all m firms send the same number of signals: $s_i = s$ for all i, $S = m\cdot s$, and $\mu(s_i, S_{-i})$ becomes $\hat{\mu}(s) = \mu(s, (m-1)\cdot s)$.

Claim 3.2: $\hat{\mu}(s)$ is a one-to-one function of s, with

$\hat{\mu}(0) = 0$,

$0 \leq \Delta\hat{\mu}(s)/\Delta s \leq 1$,

$\Delta^2\hat{\mu}(s)/\Delta s^2 < 0$, and

$\lim_{s\to\infty} \hat{\mu}(s) = n/m$

Proof: See appendix A.

Thus, if all firms send an infinite number of signals they may expect to share equally the whole aggregate demand. Note that for m=1 we describe the case of a monopolist.

The claims of this section are illustrated in figure 3.1.

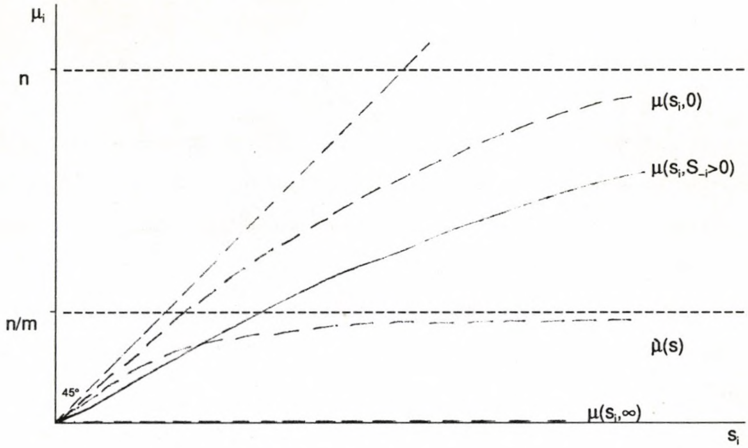

fig. 3.1 expected demand

3.3 Optimization and Equilibrium

Now we are in the position to consider firm i's optimization problem. As a strategy t_i for firm i is a pair (z_i, s_i) the first-order conditions (FOCs) for maximization of firm i's payoff are a system of two equations:[15]

$$\Delta V_i(z_i, s_i, S_{-i})/\Delta s_i = \Delta R_i(z_i, s_i, S_{-i})/\Delta s_i - \Delta K(s_i)/\Delta s_i = 0 \qquad [3.2]$$
$$\Delta V_i(z_i, s_i, S_{-i})/\Delta z_i = \Delta R_i(z_i, s_i, S_{-i})/\Delta z_i - \Delta C(z_i)/\Delta z_i = 0$$

Claim 3.3 a: $\Delta R_i/\Delta s_i = p \cdot F[z_i-1] \cdot Pr(S)$, where F[z] denotes $\Sigma_{q=0}^{z} f[q]$

b: $\Delta R_i/\Delta z_i = p \cdot (1 - F[z_i])$

Proof: See appendix B.

In other words, the gross revenue for firm i of sending one additional signal, given the strategies of the other firms, is the price p multiplied by the probability that firm i would have had still at least one unit of the commodity available multiplied by the probability that this additional signal will lead to a consumer visiting firm i. And the gross revenue for firm i of supplying one additional unit of the commodity, given the strategies of the other firms, is the price p multiplied by the probability that it would have sold out otherwise.

It is advantageous for firm i to increase its signaling s_i with one unit as long as $\Delta R_i/\Delta s_i > \Delta K/\Delta s_i = k$. Similarly, it is advantageous for firm i to increase its supply z_i with one unit as long as $\Delta R_i/\Delta z_i > \Delta C/\Delta z_i$.

We are interested only in a SNE. Therefore, having derived the FOCs for maximization of firm i's payoff, we evaluate these conditions only for those cases in which each firm chooses the same strategy. Hence, $z_i = z$ and $s_i = s$ for all i, S = m·s, and by FOC$^+$ we denote a first-order-plus-symmetry condition.

Claim 3.4 a: For every value of z there exists exactly one value of s, denoted by s(z), for which the first FOC$^+$ is satisfied. This function is characterized by s(0) =

0, $\Delta s(z)/\Delta z \geq 0$, $\lim_{Z\to\infty} s(z) = s^{max} = \{s: \Delta R_i(z{=}\infty, s)/\Delta s_i = k\}$, and $s(z) \geq z$ for all z as long as $s(z) < s^{max}$. Moreover, $s^{max} > 0$ if and only if $n/N > k/p$.

b: For every value of s there exists exactly one value of z, denoted by z(s), for which the second FOC^+ is satisfied. This function is characterized by $z(0) = 0$, $\Delta z(s)/\Delta s \geq 0$, $\lim_{S\to\infty} z(s) = z^{max} = \{z: \Delta R_i(z, s{=}\infty)/\Delta z = \Delta C/\Delta z\}$, and $z(s) \leq s$ for each s. Moreover, if $\Delta^2 C/\Delta z^2 \geq 0$ for all z then $z^{max} > 0$ if and only if $n/m > -\ln\{1 - \{\Delta C(0)/\Delta z\}/p\}$.[16]

Proof: See appendix B.

In figure 3.2 both curves are drawn. Clearly, if a firm does not produce it doesn't signal either, and the other way round. Moreover, there is a maximum level of signaling, which is related to the fact that beyond that level it is very unlikely that the receiver of an additional signal will respond to that signal. Thus, whatever the level of production the expected gains from an additional signal are below its costs. Similarly, there is a maximum level of production, which is related to the fact that it is very improbable that there will ever come a customer to buy it, whatever the level of signaling.

At a point of intersection of the two curves both FOCs for maximization of firm i's payoff are fulfilled, while each firm chooses the same strategy $\hat{t}$. Thus, such a point may represent a SNE.

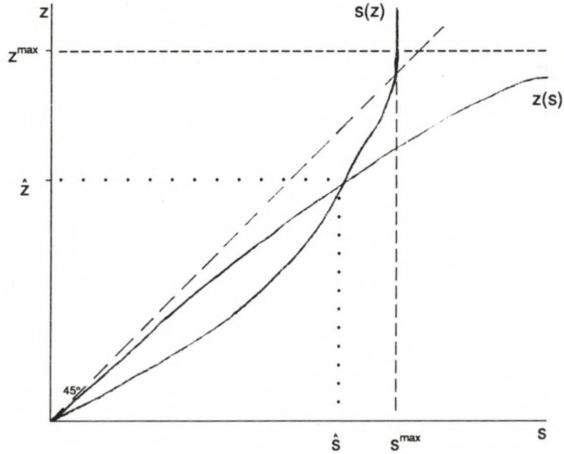

fig. 3.2 first-order-plus-symmetry condition

Now, we turn to the second-order condition (SOC) for $\hat{t}$ to be a SNE strategy:

$\Delta^2V_i(z_i, s_i, S_{-i})/\Delta z_i^2 \cdot \Delta^2V_i(z_i, s_i, S_{-i})/\Delta s_i^2 - \{\Delta\{\Delta V_i(z_i, s_i, S_{-i})/\Delta s_i\}/\Delta z_i\}^2 > 0$

and $\Delta^2V_i(z_i, s_i, S_{-i})/\Delta s_i^2 < 0$

The following claim gives a sufficient condition for this to be satisfied, given a strategy $\hat{t}$ at which the FOC$^+$s are fulfilled.

Claim 3.5: If $\{\Delta^2C(\hat{z})/\Delta\hat{z}^2\}/p > f[\hat{z}+1]/\hat{z}$ then the SOC is fulfilled.

Proof: See appendix B.

Corollary 3.1: If the SOC is fulfilled then necessarily $\Delta^2C(\hat{z})/\Delta\hat{z}^2 > 0$.

Proof: This follows directly from the fact that $p > 0$, $f[\hat{z}+1] > 0$ and $\hat{z} > 0$. □

Thus, a necessary condition concerning the production technology is that there are decreasing returns to scale (at least locally). The economic meaning of the sign of the second derivative of the cost function is clear, but, a priori, it doesn't seem to make much sense to make further assumptions concerning the shape of the C(z) function, i.e. with respect to the third derivative. Note that whether the SOC will actually be fulfilled depends also upon the strategy $\hat{t}$ for which the FOC⁺s are fulfilled.

Suppose that there were no market uncertainty. That is, when firm i supplied z units it would know that it would sell z units: $f[z] = 1$. Then, $f[z+1] = 0$, and the condition of claim 3.5 would be $\Delta^2 C(z)/\Delta z^2 > 0$, which is a rather familiar expression for models without market uncertainty.

Finally, one has to consider the payoff V_i to firm i. Clearly, if the expected profit when all firms choose strategy t is negative, firm i will prefer to stay inactive, and no strictly positive SNE exists.

Proposition 3.2: Necessary, but not sufficient, conditions concerning the parameter values for a SNE to exist are:

$n/N > k/p$,

$n/m > -\ln\{1 - \{\Delta C(0)/\Delta z\}/p\}$, and

$\Delta^2 C(z)/\Delta z^2 > 0$

Proof: See claim 3.4 and corollary 3.1. □

These conditions imply that n, p and $\Delta^2C(z)/\Delta z^2$ must be large enough, while N, m, k and $\Delta C(0)/\Delta z$ must be small enough, as would have been the intuition. However, the conditions are not sufficient. We have done a numerical analysis in order to determine the values of k, C(.), p, n(.), m and N for which a SNE exists. The details and results can be found in appendix D.

We can summarize the findings as follows. A SNE won't exist when the costs of making a market and/or producing for the market are too high relative to the price p for each possible extent of the market at all levels of production. This may be due not only to the level of prices (p) and costs (k and $\Delta C/\Delta z$) as such, but also to a too small number of potential buyers (n) in the economy or a too large number of competing firms (m).

In the next section on comparative statics we give an analysis of the effects of changes in the parameter values.

3.4 Comparative Statics

We now consider more detailed the influence of parameter changes. This leads to the following series of claims. First, we concern the FOC$^+$s, referring to the s(z) and z(s) curves of figure 3.2, and then the condition V>0. We assume that the necessary conditions of proposition 3.2 are fulfilled.

Claim 3.6 a: There is a number r, r > 0, such that for any k < r the FOC$^+$s are fulfilled for some strategy $\hat{t}$.

b: There is a number r, r > 0, such that for any $\Delta C(z)/\Delta z$ < r for all z the FOC$^+$s are fulfilled for some strategy $\hat{t}$.

Proof: See appendix B.

The influence of the parameter p clearly depends upon the price–elasticity of demand $\varepsilon = -dn(p)/dp \cdot p/n(p)$.

Claim 3.7 a: $\Delta s(z)/\Delta p \leq 0$ if $\{F[z-1] - \mu \cdot f[z-1]\} \geq 0$

and $\varepsilon \geq F[z-1]/\{F[z-1] - \mu \cdot f[z-1]\}$

while $\Delta s(z)/\Delta p \geq 0$ otherwise

b: $\Delta z(s)/\Delta p \geq 0$ if $\varepsilon \leq (1 - F[z])/(\mu \cdot f[z])$

while $\Delta z(s)/\Delta p \leq 0$ otherwise

Proof: See appendix B.

The effect of a change in p if aggregate demand n(p) would be insensitive to price changes is considered in the following corollary.

Corollary 3.2: If $\varepsilon = 0$ then there is a number r, $r > 0$, such that for $p > r$ the FOC's are satisfied for some strategy $\hat{t}$.

Proof: As long as p increases the s(z) curve in figure 3.2 shifts to the right and the z(s) curve upwards. Hence, at a certain point they must intersect. □

Claims 3.6 and 3.7 and the corollary concern the net gains per transaction. If these gains are not high enough, no markets will be created. However, each firm's gains also depend upon its trading opportunities. The following claim considers the importance of the numbers of potential customers and competing firms.

Claim 3.8 a: $\Delta s(z)/\Delta(n/m) \geq 0$ if $\{F[z-1] - \mu \cdot f[z-1]\} \geq 0$

$\Delta s(z)/\Delta(n/m) \leq 0$ otherwise

b: $\Delta z(s)/\Delta(n/m) \geq 0$

Proof: See appendix B.

Thus, the direct effect of a change in the parameters m and those of n(.) is ambiguous. The problem is that there are two opposing effects upon the s(z) curve in figure 3.2. On the one hand, if the probability of success of any given signal (Pr(s)) increases, then the probability of success of an additional signal increases. However, on the other hand, also the probability of success of all other signals increases, implying that the firm may expect more visitors and the probability that the firm would have at least one unit of the commodity left (F[z−1]) decreases, and thus the probability that an additional visitor doesn't make sense increases. However, at least, we know what happens when n/m approaches infinity.

Claim 3.9: If $n/m \rightarrow \infty$ then the FOC$^+$s will be fulfilled for $\hat{t} \equiv (\hat{z} = \{z: \Delta C/\Delta z = p-k\}, \hat{s} = \hat{z})$

Proof: See appendix B.

Thus, when n/m goes to infinity, each signal sent will surely lead to a consumer visiting its sender, and hence a situation of certainty is approached.

Next, we consider the influence of parameter value changes upon the expected profit.

Claim 3.10 a: $dV_i/dk < 0$

b: $dV_i/d(\Delta C/\Delta z) < 0$

c: $dV_i/dp > 0$ if $\varepsilon < 1 + \{z \cdot (1 - F[z])\}/(\mu \cdot F[z-1])$

$dV_i/dp \leq 0$ otherwise

d: $dV_i/d(n/m) > 0$

Proof: See appendix B.

Thus, if the FOC$^+$s and the condition V>0 were not fulfilled for a given set of parameters, they may be fulfilled form some $k' < k$, some $(\Delta C/\Delta z)' < (\Delta C/\Delta z)$ for all z, some $p' > p$, or some $(n/m)' > (n/m)$.

As long as one doesn't further restrict the cost function C(z), it is not possible to say much about the effect of parameter changes upon the SOC. One can only observe that $f[\hat{z}+1]$ is bounded below 1, and that therefore the right-hand side of the equation of claim 3.5 approaches zero if $\hat{z}$ goes to infinity, implying that it might be more likely that the SOC is fulfilled when $\hat{z}$ is larger.

Corollary 3.3: if a SNE (z^*, s^*, V^*) exists for a given 'set of parameters' {k, C(.), p, N, n(.), m} then

a: if k decreases then z^*, s^* and V^* increase

b: if $\Delta C/\Delta z$ is smaller for all z then z^*, s^* and V^* increase

c: if p increases and $\varepsilon = 0$ then z^*, s^* and V^* increase

d: if n(p)/m increases then V^* increases, while the effect upon z^* and s^* may be positive or negative.

Proof: See claims 3.6, 3.8 and 3.10, and corollary 3.2. □

Thus, higher net gains per transaction will, ceteris paribus, lead to a SNE with increased supply, increased market-making activity and increased profits. A decrease of market uncertainty will lead to higher profits, but not necessarily to increased signaling and production activity, as there is no unambiguous relation between the number of firms per interested consumer and the amount of market-making activity.

3.5 Some Characteristics of the SNE

In a SNE the economy splits up in a number of possibly overlapping markets: each firm creates its own market of size s^*, and produces z^*. Supposing that a SNE exists, we now analyze its characteristics.

Sometimes a relation between *the division of labor* and *the extent of the market* is suggested; the latter determining the first (Smith [1776]) or the other way round. Take the number of firms m, for given N, as a measure of the division of labor, and the aggregate number of signals S as a measure of the extent of the market.

Proposition 3.3: The extent of the market (S) is a function of the division of labor (m), and the sign of $\Delta S/\Delta m$ is not a priori determined.

Proof: Given the set of parameters, individual firm choose z and s. S is just the aggregate market-making activity: $S = m \cdot s$. Hence, $\Delta S/\Delta m = s + m \cdot \Delta s/\Delta m$. As shown in claim 3.8 the value of $\Delta s/\Delta m$ is not yet determined. □

Thus, in this model the extent of the market is determined by the division of labor, but the consequence of a small change of the latter for the first is not clear.

Proposition 3.4: If the 'set of parameters' $\{k, C(.), p, N, n(.), m\}$ leads to a SNE (z^*, s^*, V^*) then the 'set of parameters' $\{k, C(.), p, \alpha \cdot N, \alpha \cdot n(.), \alpha \cdot m\}$ leads to exactly the same SNE (z^*, s^*, V^*) for any $\alpha > 0$.

Proof: See appendix C.

In other words, the SNE and individual market outcomes are independent of the size of the economy as long as the proportions of types of agents, i.e. firms and (interested) consumers, remain constant.[17] Hence, we could interpret the parameters of the model such that the number of agents N is countably infinite, while m and n(.) are the fractions of firms and interested consumers in the population.
Until here we just supposed that only firms may send signals. The following proposition states which assumption suffices to make that supposition right.

Proposition 3.5: Sufficient to get as an analytical result that consumers do not signal is to assume: $m/N < k/\{U_i(1, \omega_i - p) - U_i(0, \omega_i)\}$ for each consumer i.

Proof: Analogous to part of the proof of claim 3.4.

This proposition makes clear why, usually, consumers do not create buyers' markets, even when the same market-making technology is available to them. Consumers are interested in buying only a very limited number of units, in our model only 1, while firms usually want to sell much more units, focussing upon V_i . And related to this is the fact that, in general, there are much more consumers (n) of a certain commodity than firms (m) selling that commodity, making it more difficult for consumers to find firms than the other way round.

To analyze markets one often uses the concepts of demand and *supply functions*. A supply function of a firm represents the firm's willingness to supply as a function of the input and output prices, taking into account its production technology. It is a purely individual characteristic of the sellers, and is independent of the buyers willingness to buy (see e.g. Varian [1984]).

Proposition 3.6: In this non-Walrasian setting no relevant supply function exists.

Proof: See appendix C.

The point is that, in this non-Walrasian setting, what is relevant is a firm's effective supply z. This effective supply cannot be determined independently of the firm's trading opportunities, i.e. of the stochastic demand directed to it. The expected value of this demand depends upon the aggregate demand in the economy n(p).

Now we consider the efficiency of the SNE. Clearly, the allocation mechanism as such is informationally inefficient. We focus upon efficiency given the trading and communication structure of the model (see Ulph & Ulph [1975]).

One of the attractive features of Walrasian models is that The Market is operated very efficiently. A market is efficient if all mutually advantageous trades are carried out, which implies that one will not find rationed demanders and rationed suppliers at the same time (see Benassy [1982]). In this sense the market outcome of a SNE is inefficient.

Proposition 3.7: Prob $[(x_i - z_i) \cdot (x_j - z_j) < 0] > 0$ for each pair i, j where $i \in B$, $j \in D$ and $z > 0$.

Proof: See appendix C. Here we give a sketch. Firm i's supply is z^*, while the stochastic demand directed to it is represented by f[q]. Thus, the probability that firm i is rationed is equal to the probability that it will receive less than z^* buyers:
Prob $[(x_i - z_i) > 0] = F[z^* - 1] > 0$.
An interested consumer j has a unit demand and may visit only one firm. Buyers will be rationed when they don't receive any signal or when they visit a firm which has sold out already. In appendix C we see that both possibilities may occur with positive probabilities: Prob $[(x_j - z_j) < 0] > 0$. □

Thus, in a SNE the overall economy will not be orderly for each p as there may be some buyers as well as some firms rationed at the same time.[18] [19] This seems to be a rather important characteristic of a decentralized economy.
As rationing with respect to the consumers is all-or-nothing, from their point of view the probability to be rationed is a good measure of the performance of the economy. Firms, however, do take the market uncertainty already into account when deciding upon their effective supply, and rationing is a quite 'natural' affair for them. Therefore, we also consider another measure of efficiency concerning the firms.
Until here we considered the non-cooperative equilibrium concept of a SNE. We can compare this with an equilibrium which would be in the joint interest of all firms.

Definition 3.3: A Symmetric Cooperative Equilibrium (SCE) is a vector of strategies $t^c \equiv (t^c_1, .., t^c_i, .., t^c_m)$ such that $t^c_i = t^c_j$ for each i, j, and the sum of the payoffs of all firms is maximized by choosing strategy t^c for each firm, i.e. $\sum V_i(\mathbf{t}^c) \geq \sum V_i(\mathbf{t})\ \forall$ **t**.

Proposition 3.8: The equilibrium strategy t^* of a SNE involves more communication and production, but lower profits than the equilibrium strategy t^c of a SCE, i.e. $z^* > z^c$ and $s^* > s^c$, while $V^* < V^c$.

Proof: See appendix C.

Thus, a SNE is not efficient from the firms' point of view in the sense that a better, i.e. preferred by all firms, vector of strategies exists. However, each individual firm will have an incentive to deviate from the SCE strategy t^c. Moreover, consumers are worse of in a SCE.

Proposition 3.9: Prob [cons. rationed| SNE] < Prob [cons. rationed| SCE]

Proof: See appendix C.

To conclude the characterization of the SNE, we give some numerical examples. In the first six rows of table 3.1 only the 'parameters' m and n vary. If we compare, for example, row 3 with row 6, we see that when the relative number of firms and consumers (n/m) doesn't change the SNE remains the same. Interesting is also a comparison of row 4 with 5, where the number of firms (m) increases enormously. Nevertheless, in this example, each firm's market-making signaling activity does not

m	n	c	k	z^*	s^*	V^*	Prob [firm rat.]	Prob [cons. rat.]	Prob [neg. prof.]
5	5,000	.001	.010	657	22,053	220.0	.33	.35	.00
100	5,000	.001	.010	61	4,641	1.1	.94	.01	.45
100	10,000	.001	.010	112	8,742	5.8	.87	.01	.26
10	37,500	.001	.010	967	3,196	470.0	.03	.74	.00
750	37,500	.001	.010	61	4,640	1.6	.93	.00	.42
750	75,000	.001	.010	112	8,740	5.8	.87	.01	.26
100	10,000	.010	.050	34	389	5.7	.60	.69	.12
100	10,000	.001	.050	85	1,323	3.3	.90	.27	.33
100	10,000	.010	.010	76	1,762	29.0	.21	.25	.00
100	10,000	.001	.010	112	8,742	5.8	.87	.01	.26
100	10,000	.010	.005	84	2,754	35.0	.15	.17	.00
100	10,000	.001	.005	112	17,480	5.8	.87	.01	.26

Table 3.1 some examples

decrease but even increases, while its supply and profit fall dramatically. As the last three columns show, the economic situation changes from highly favorable for the firms to highly advantageous for the consumers.

In the last six rows the parameters c and k vary. Notice that the probability of a firm being rationed is quite high on average, while the probability of negative profits is much more moderate.

4. Conclusion

4.1 Further Possibilities of the Framework

The basic features of a decentralized economy taken into account in our model are, that the behavior of individual agents is based upon some knowledge about possible transaction opportunities, that agents have a very limited knowledge of their economic environment, that such knowledge requires some kind of communication or interaction between agents, and that agents face uncertainty as to their immediate trading opportunities. A number of problems concerning decentralized trade is related to these features, and may therefore very well be studied within this framework.

For example, one could analyze the existence of *central markets*. Suppose that firms may decide to 'cooperate' physically in the market-making process. Instead of each firm selling its production in its own market, there may be one or more common, central markets or central distribution points. Suppose that the technology of market-making is still the same (the central distributors sending signals giving the address of the distributor and the message that he sells the commodity), and that there are no additional costs of running a central market. When we assume that each firm may sell in a common market proportionally to its contribution in the market-making costs, one could, for example, consider the non-cooperative solution concept of a Nash equilibrium. Each firm chooses a market to join and decides how much to contribute to the signaling activity, taking the choices of the other firms as given. This seems to capture the essential function of a central distributor. Notice that if all firms would decide to join the same central market, we get a situation which reminds somewhat the situation sketched in figure 1.1 concerning a Walrasian model.

Also the functioning of *middlemen* may be very well analyzed within the framework of our model. Middlemen are not intrinsically interested in the commodity itself, i.e. they belong neither to the firms nor to the consumers. They buy from firms sellers and sell to buyers. This description still leaves room for a number of functions of middlemen (e.g. reducing real transaction costs, reducing storing costs, forming a buffer between fluctuating demand and supply, speculation, etc.), but the distinguishing characteristic of middlemen is that they make a profit by taking account of the matching problem of the economy. Thus, they create markets by sending signals to get contact with both firms and consumers.

The model seems to be an interesting starting-point to study *price-making* in a decentralized economy. The resulting market structure in a SNE is imperfectly competitive, although the commodity traded is homogeneous. Each firm signals to s agents, by which it creates its own market. Thus, buyers may know that a firm finds as a maximum s alternative buyers in the market, and therefore buyers have some monopsony power. Each buyer on the other hand, can only trade with those firms of which he received a signal. Thus, each firm may know that a buyer visiting him will know only a limited number of alternative firms to visit, and therefore they will have some monopoly power. In order to study price-making, some of the simplifying assumptions should be relaxed, but then the analysis becomes quite complicated (see e.g. Kormendi [1979]). Suppose e.g. that buyers may make more visits, knowing the distribution of prices at the beginning of the period, but not knowing which firm asks which price. In their search decision buyers do not only compare the direct costs of visiting another firm (which are zero in our model) with the probability to find a lower price (as they do in standard search models), but they also have to take into account the probability that they won't be able to find the commodity for an acceptable price at all in this period if they search too long. Moreover, during the search process there is uncertain recall (visited firms may sell out), while the distribution of prices may be changing (probably cheaper firms sell out more easily).

Finally, the model of decentralized trade proposed might be useful for a study of the phenomenon of *liquidity*: an asset being more liquid if it may be sold cheaper and more surely (see Hahn [1988]), or problems like *effective demand failures*: an agent not willing to demand/supply more of one commodity because of uncertainty about his trading possibilities concerning another commodity (see Grandmont [1988]).

4.2 Discussion

Clearly, we have made a number of abstractions from reality in this model. For example, most real world commodities are not homogeneous. But homogeneity and the lack of uncertainty about qualitative aspects may be defended as an abstraction in order to focus upon other aspects of decentralized trade.

Next, the information transmission mechanism might be too specific. For example, agents may hear from friends about the newest shops in town, buyers may use the Yellow Pages to find a seller, or they may randomly visit shops, etc. However, basically, these possibilities seem to fit rather well in the signaling framework used. In the first case, one could consider these friends as sending signals or one could assume that each signal sent by a firm reaches one agent plus possibly some friends of his. Concerning the second case, one may consider a decision to be inserted in the list of the Yellow Pages as sending some signals. And about the last case, randomly visiting shops implies that these sellers have signaled already that they sell a certain commodity by having furnished a shopwindow etc. Thus, signaling consists of all the possible ways in which an agent may make information about his own type known to some other agents and is not restricted to e.g. advertising in a strict sense.

More important is the lack of any relation between information and market experiences of different periods. This implies that the model fits best to those commodities which are purchased rather infrequently. Examples of such commodities can be found in the retailing of consumer durables, the transportation sector (e.g. airline tickets), or the industry of leisure and entertainment (e.g. hotels, restaurants, theaters).

However, a lot of commodities are bought on a repeated basis, and it is not realistic to assume that agents start from scratch in each period. In a dynamic model agents will remember some information about their economic environment from previous periods (even when they have a bounded memory). Moreover, agents' own market experience in the past will carry some weight when making new decisions. Thus, for example, firms may try to target their signals to interested consumers. And consumers will not choose a firm at random, as these firms will build up a reputation. Rather, consumers might opt for patronizing a certain firm as long as their market experiences with that firm are positive enough.

Hence, while in our model the number of information and trading links between agents is endogenous, in a dynamic model also the choice of the specific links should be made endogenous. Moreover, although we focussed upon the limited knowledge of the economic environment by individual agents, we still assumed that firms did have objective knowledge about the aggregate demand and the aggregate signaling activity. In a dynamic model this should be replaced by their subjective perception based upon their own experiences.

One of the interesting features of the SNE described that would come into its own only in a dynamic model, is that even when ex-ante, expected market experiences are equal for all agents of the same class, actual market experiences may differ widely among agents. Take, for example, the payoff V to the firms. Clearly, if $V_i < 0$, i.e. expected profits are negative, firm i will prefer to stay inactive. But we have not analyzed what will happen then. We put the existence conditions of a SNE in terms of parameter values n(.) and m, but that we did not talk about exit/entry of firms. The reason is that this doesn't seem to make much sense in a static model and an analysis that does not go beyond symmetric equilibria. Moreover, even if $V_i \geq 0$ the possibility exists that ex-post, actual profits of firm i are negative. Hence, in a dynamic model also problems as bankruptcy would enter the scene.

Appendix A: Stochastic Demand

Proof proposition 3.1: Suppose that firm i has sent s_i signals, and consider just one of those signals. Firm i sends this signal at random into the population. (Technically, suppose firm i has put all agents in an urn, draws just one agent to determine the destination of the signal, and replaces that agent). The probability that this signal from firm i is received by an interested consumer is n(p)/N. Each interested consumer puts all received signals in a urn and draws just one signal out of his urn. Supposing that the signal from firm i is received by an agent who has in total received x signals from various firms, the probability that he would draw firm i's signal is 1/x. Thus, one has to determine x, the number of signals received by any given agent.

From the point of view of such an agent, the destination of each signal sent in the economy is the outcome of a Bernoulli trial with two possible outcomes: the signal will reach him or another agent. Thus, the number of signals received by any given agent has a binomial distribution with parameters the total number of signals sent and the probability of reaching this given agent. If the number of Bernoulli trials in the sequence is large and the probability of reaching the given agent is close to 0, the binomial distribution may be approximated by a Poisson distribution (see DeGroot [1986]). A glance at the appropriate probability tables suggests that such an approximation is reasonable when the number of trials is greater than 25, while the probability of success is smaller than 0.1. We assume that the sets B and D are such that both conditions will be fulfilled. Thus, the probability that this signal from firm i is received by an agent who has got (x−1) other signals is:

$$\text{Prob [x−1 other signals]} = e^{-\lambda} \cdot \frac{\lambda^{x-1}}{(x-1)!}$$

where λ = S/N is the expected number of signals received by any given agent

with 1/N = Prob ['hitting' any given agent]

S = aggregate number of signals sent by all firms

Hence, the probability that any given signal from firm i will lead to an interested consumer visiting firm i is:

$$\begin{aligned} \Pr(s) &= (n(p)/N) \cdot \textstyle\sum_{x=1}^{\infty} (1/x) \cdot e^{-\lambda} \cdot \dfrac{\lambda^{x-1}}{(x-1)!} \\ &= (n(p)/N) \cdot (1/\lambda) \cdot \textstyle\sum_{x=1}^{\infty} e^{-\lambda} \cdot \dfrac{\lambda^{x}}{x!} \\ &= (n(p)/N) \cdot (1/\lambda) \cdot (1 - e^{-\lambda}) \end{aligned}$$

$= (n(p)/S) \cdot (1 - e^{-S/N}) \;\; \forall\, i$

This probability Pr(S) refers to one single signal sent. From the point of view of firm i, each signal it has sent is a Bernoulli trial with two possible outcomes: the receiver will visit firm i or not. The sum of a sequence of s_i of such Bernoulli trials is a random variable which has a binomial distribution with parameters s_i and Pr(S). (Here we make a small error. That is, when a buyer has received more than one signal from firm i, he will perform only one Bernoulli trial.) If the number of signals is large while the probability that the receiver will visit firm i is close to 0, the number of buyers visiting firm i may be approximated by a Poisson distribution with parameter $\mu_i = \mu(s_i, S_{-i}) = s_i \cdot Pr(S) = (s_i/S) \cdot n(p) \cdot (1 - e^{-S/N})$. As each visiting agents demands exactly one unit, the demand q_i facing firm i has the same Poisson distribution: $f[q_i|\, \mu(s_i, S_{-i})]$.□

Proof claim 3.1 a: $\mu(s_i, S_{-i}) = s_i \cdot Pr(S)$, with $Pr(S) = (n/S) \cdot (1 - e^{-S/N})$ and $S = s_i + S_i \Rightarrow \Delta\mu(s_i, S_{-i})/\Delta s_i = Pr(S) + s_i \cdot \Delta Pr(S)/\Delta s_i$

The last term of the right-hand side of this equation is the indirect effect of sending one additional signal by firm i. This indirect effect is negative as $\Delta Pr(S)/\Delta s_i = -(n/S^2) \cdot \{1 - e^{-S/N} \cdot (1 + S/N)\} < 0$. That is, each signal sent becomes a little bit less likely to be successful when an additional signal competes with it. However, as long as S is relatively large, this indirect effect is negligible from the point of view of firm i: $\lim_{S \to \infty} \Delta Pr(S)/\Delta s_i = 0$.

The direct effect is the probability that any given signal sent will lead to its receiver visiting the sender of the signal. As all probabilities: $0 \leq Pr(S) \leq 1$.

Hence, $\Delta^2\mu(s_i, S_{-i})/\Delta s_i^2 = \Delta Pr(S)/\Delta s_i < 0$ (see above).

$$\lim_{s_i \to \infty} \mu(s_i, S_{-i}) = \lim_{s_i \to \infty} \{s_i \cdot n \cdot (1 - e^{-(s_i + S_{-i})/N})\}/(s_i + S_{-i}) = \infty/\infty$$

Applying L'Hôpital's Rule gives:

$$\lim_{s_i \to \infty} n \cdot \{(1 - e^{-(s_i + S_{-i})/N}) + s_i/(N \cdot e^{-(s_i + S_{-i})/N})\} = n \cdot (1 + \infty/\infty)$$

Applying L'Hôpital's Rule once again for the last quotient leads to:

$$\lim_{s_i \to \infty} \mu(s_i, S_{-i}) = n \cdot (1 + \lim_{s_i \to \infty} e^{-(s_i + S_{-i})/N}) = n \quad \square$$

b: $\Delta\mu(s_i, S_{-i})/\Delta S_{-i} = \Delta Pr(S)/\Delta S_{-i}$. As $S = s_i + S_{-i}$, $\Delta Pr(S)/\Delta S_{-i}$ $= \Delta Pr(S)/\Delta s_i < 0$ (see above).

$$\lim_{s_i \to \infty} \mu(s_i, S_{-i}) = \lim_{s_i \to \infty} s_i/(s_i + S_{-i}) \cdot n \cdot (1 - e^{-(s_i + S_{-i})/N}) = 0 \quad \square$$

Proof claim 3.2: $\hat{\mu}(s) = n/m \cdot (1 - e^{-m \cdot s/N})$. To get $\hat{\mu}(0)$ and $\lim_{s \to \infty} \hat{\mu}(s)$ just substitute s.

$\Delta\hat{\mu}(s)/\Delta s = (n/N) \cdot e^{-m \cdot s/N} \Rightarrow 0 \leq \Delta\hat{\mu}(s)/\Delta s \leq 1$

$\Delta^2\hat{\mu}(s)/\Delta s^2 = -(n \cdot m/N^2) \cdot e^{-m \cdot s/N} < 0 \quad \square$

Appendix B: Optimization

Proof claim 3.3 a: We rewrite $R_i(z_i, s_i, S_{-i})$ by omitting subscripts and arguments as much as possible for notational convenience, and observing that

$$\sum_{q=0}^{z} q \cdot f[q] = \sum_{q=1}^{z} q \cdot e^{-\mu} \cdot \frac{\mu^q}{q!} = \mu \cdot \sum_{q=1}^{z} e^{-\mu} \cdot \frac{\mu^{q-1}}{(q-1)!}$$

$$= \mu \cdot \sum_{y=0}^{z-1} e^{-\mu} \cdot \frac{\mu^y}{y!} = \mu \cdot F[z-1]$$

Hence, $R_i = p \cdot \{\mu_i \cdot F[z_i-1] + z_i \cdot (1 - F[z_i])\}$

$$\Delta R_i/\Delta s_i = p \cdot \{\Delta\mu_i/\Delta s_i \cdot F[z_i-1] + \mu_i \cdot \Delta F[z_i-1]/\Delta\mu_i \cdot \Delta\mu_i/\Delta s_i$$
$$- z_i \cdot \Delta F[z_i]/\Delta\mu_i \cdot \Delta\mu_i/\Delta s_i\}$$

Now, $\Delta F[z]/\Delta\mu = F[z-1] - F[z] = -f[z]$ and $\Delta\mu/\Delta s = Pr$ (see claim 3.1).

Hence, $\Delta R_i/\Delta s_i = p \cdot \{F[z_i-1] - \mu_i \cdot f[z_i-1] + z_i \cdot f[z_i]\} \cdot Pr$

As $f[z-1] = (z/\mu) \cdot f[z]$, we get $\Delta R_i/\Delta s_i = p \cdot F[z_i-1] \cdot Pr$. □

b: Again, we first rewrite $R_i(z_i, s_i, S_{-i})$.

$$R = p \cdot \{ \textstyle\sum_{q=0}^{z} q \cdot f[q] + z \cdot (1 - F[z])\}$$

Summing up the first term by parts and rewriting the second term gives

$$R = p \cdot \{z \cdot F[z] - \textstyle\sum_{q=0}^{z-1} F[q] + z - z \cdot F[z]\}$$
$$= p \cdot \{z - \textstyle\sum_{q=0}^{z-1} F[q]\}$$

Then, $\Delta R_i/\Delta z_i = p \cdot \{1 - F[z_i]\}$ □

Proof claim 3.4 a: The FOC is: $\Delta R_i(z_i, s_i, S_{-i})/\Delta s_i = \Delta K(s_i)/\Delta s_i$. In case of symmetry, $z_i = z$, $s_i = s$ and $S_{-i} = (m-1) \cdot s$ for all i. Hence, we get $\Delta R_i(z, s)/\Delta s_i = \Delta K(s)/\Delta s_i \Rightarrow p \cdot F[z-1] \cdot Pr(s) = k$. First, we keep constant z. If $s = 0$ then $\hat{\mu}(s) = 0$ and hence $F[z-1] = 1$ for all $z \geq 1$. $\lim_{s\downarrow 0} Pr(s) = \lim_{s\downarrow 0} n/(m \cdot s) \cdot (1 - e^{-m \cdot s/N}) = 0/0$. Applying L'Hôpital's Rule gives: $\lim_{s\downarrow 0} (n/N) \cdot e^{-m \cdot s/N} = n/N$.

Hence, $\Delta R_i(z, s{=}0)/\Delta s_i = p \cdot n/N$ for all z.

$\Delta\{\Delta R_i(z, s)/\Delta s_i\}/\Delta s = p \cdot \{\Delta F[z-1]/\Delta\mu \cdot \Delta\mu/\Delta s \cdot Pr(s) + F[z-1] \cdot \Delta Pr(s)/\Delta s\} < 0$ as the only negative terms are $\Delta F[z-1]/\Delta\mu$ and $\Delta Pr(s)/\Delta s$ (see claims 3.1 and 3.3).

$\lim_{s\to\infty} Pr(s) = \lim_{s\to\infty} n/(m \cdot s) \cdot (1 - e^{-m \cdot s/N}) = 0$.

Hence, $\lim_{s\to\infty} \Delta R_i(z, s)/\Delta s_i = \lim_{s\to\infty} p \cdot F[z-1] \cdot Pr(s) = 0$.

Now, we consider the variable z. $\Delta R_i(z{=}0, s)/\Delta s_i = 0$,

$\Delta\{\Delta R_i(z, s)/\Delta s_i\}/\Delta z = p \cdot f[z] \cdot Pr(s) > 0$ and $\lim_{z\to\infty} \Delta R_i(z, s)/\Delta s_i = p \cdot Pr(s)$.

We can draw this in figure B.1. We see that for given z there is an optimal value of s: s(z), with s(0) = 0, $\Delta s(z)/\Delta z > 0$ and $\lim_{z\to\infty} s(z) = s^{max} = \{s: p \cdot Pr(s) = k\}$. We see that if $p \cdot n/N < k$ then s(z) =0 for all z.

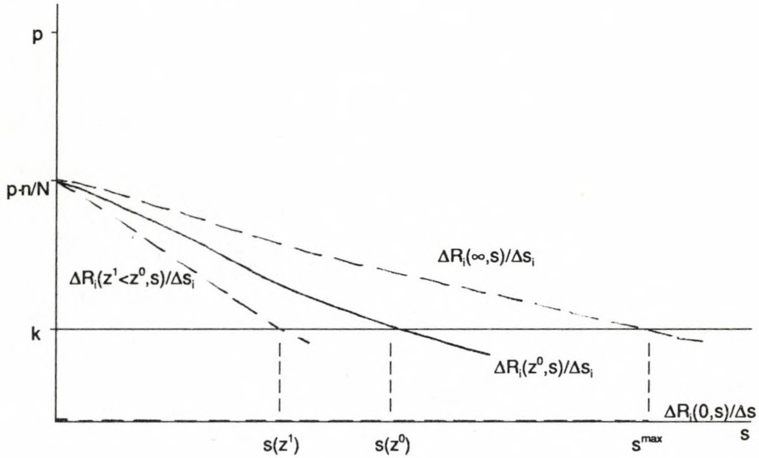

fig. B.1 FOC⁺ with respect to signaling

Finally, we have to prove that $s(z) \geq z$ for all z if $s(z) < s^{max}$. Suppose $s < s^{max}$ and $z > s$. If $z > s$ then $F[z-1] = 1$, as no firm can get more customers than the number of signals it has sent. Hence, $\Delta R_i(z, s)/\Delta s_i = p \cdot Pr(s)$. We know that if $s < s^{max}$ then $p \cdot Pr(s) > k \Rightarrow$ if $z > s$ then $\Delta R_i(z, s)/\Delta s_i > k$. Hence, for each given value of z, it will be profitable for each firm i to increase s_i with one unit as long as $s < s^{max}$ and $s < z$. Therefore, $s(z) \geq z$ for each z □

b: The FOC is: $\Delta R_i(z_i, s_i, S_{-i})/\Delta z_i = \Delta C(z_i)/\Delta z_i$. With symmetry we get $\Delta R_i(z, s)/\Delta z_i = \Delta C(z)/\Delta z_i \Rightarrow p \cdot (1 - F[z]) = \Delta C(z)/\Delta z_i$. First, we keep constant s. If z = 0 then $F[z] = e^{-\hat{\mu}(s)}$ and hence $\Delta R_i(z, s)/\Delta z_i = p \cdot (1 - e^{-\hat{\mu}(s)})$.

$\Delta\{\Delta R_i(z, s)/\Delta z_i\}/\Delta z = -p \cdot f[z+1] < 0$

$\lim_{z\to\infty} \Delta R_i(z, s)/\Delta z_i = \lim_{z\to\infty} p \cdot (1 - F[z]) = 0.$

Now, we consider the variable s. $\Delta R_i(z, s{=}0)/\Delta z_i = 0$ as $\hat{\mu}(0) = 0$, and hence $F[z] = 1$. $\Delta\{\Delta R_i(z, s)/\Delta z_i\}/\Delta s = p \cdot f[z] \cdot Pr(s) > 0$ and $\lim_{s\to\infty} \Delta R_i(z, s)/\Delta z_i =$

$p \cdot (1 - F[z| \mu{=}n/m])$ as $\hat{\mu}(\infty) = n/m$ (see claim 3.2).

The only assumption made with respect to $\Delta C(z)/\Delta z$ is that it is strictly positive. As, at this point, there is no reason to impose a particular shape of the $\Delta C(z)/\Delta z$ curve, in figure B.2 we just draw one possibility, chosen for expositional convenience.

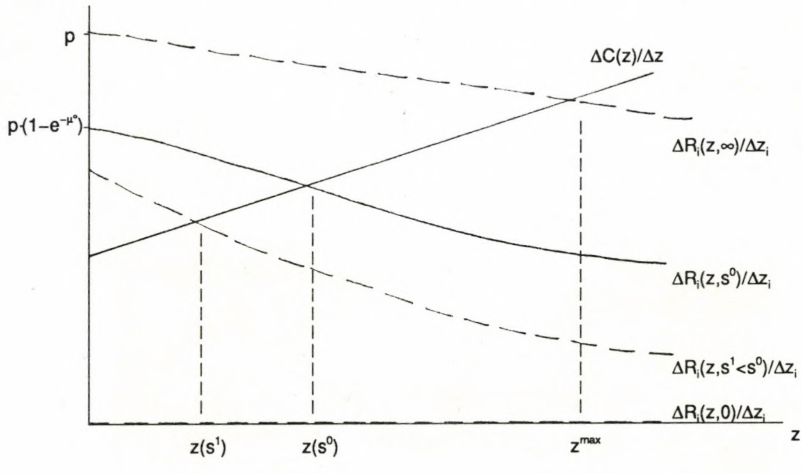

fig. B.2 FOC⁺ with respect to production

We see that there is a function z(s), with $z(0) = 0$, $\Delta z(s)/\Delta s > 0$ and $\lim_{s\downarrow\infty} z(s) = z^{max}$ $= \{z: \Delta R_i(z, s{=}\infty)/\Delta z_i = \Delta C(z)/\Delta z_i\}$.

To prove that $z(s) \leq s$, suppose $z = s$. Clearly, $F[z=s] = 1$. Hence, $\Delta R_i(z=s, s)/\Delta z_i = 0 < \Delta C(z)/\Delta z_i$.

z^{max} is given by the intersection of the $\Delta R_i(z, s=\infty)/\Delta z_i$ and the $\Delta C(z)/\Delta z_i$ curve. Hence, if $\Delta^2 C/\Delta z^2 \geq 0$ then one should have $\Delta R(z=0, s=\infty)/\Delta z_i > \Delta C(0)/\Delta z_i$, which gives $p \cdot (1 - e^{-n/m}) > \Delta C(0)/\Delta z_i$ or $n/m > -\ln\{1 - (\Delta C(0)/\Delta z_i)/p\}$. □

Proof claim 3.5: The SOC for $\hat{t}$ being the optimal strategy for firm i is:

$\Delta^2 V_i(z_i, s_i, S_{-i})/\Delta z_i^2 \cdot \Delta^2 V_i(z_i, s_i, S_{-i})/\Delta s_i^2 - \{\Delta\{\Delta V_i(z_i, s_i, S_{-i})/\Delta s_i\}/\Delta z_i\}^2 > 0$

and $\Delta^2 V_i(z_i, s_i, S_{-i})/\Delta s_i^2 < 0$

For notational convenience we omit subscripts and arguments as much as possible.

$$\Delta^2 V/\Delta z^2 = p \cdot \{(1 - F[z+1]) - (1 - F[z])\} - \Delta^2 C/\Delta z^2$$
$$= -p \cdot f[z+1] - \Delta^2 C/\Delta z^2$$
$$\Delta^2 V/\Delta s^2 = p \cdot Pr \cdot \Delta F[z-1]/\Delta\mu \cdot \Delta\mu/\Delta s - \Delta^2 K/\Delta s^2$$
$$= -p \cdot Pr \cdot f[z-1] < 0$$
$$\Delta(\Delta V/\Delta s)/\Delta z = p \cdot Pr \cdot f[z]$$

Thus, the remaining condition to check is:

$$\{-p \cdot f[z+1] - \Delta^2 C/\Delta z^2\} \cdot \{-p \cdot Pr \cdot f[z-1]\} - \{p \cdot Pr \cdot f[z]\} > 0$$
$$\Rightarrow (\Delta^2 C/\Delta z^2)/p > - f[z+1] + (f[z])^2/f[z-1]$$
$$\Rightarrow (\Delta^2 C/\Delta z^2)/p > - f[z+1] + \{f[z+1] \cdot (z+1)/\mu\}^2/\{f[z+1] \cdot (z/\mu) \cdot (z+1)/\mu\}$$
$$\Rightarrow (\Delta^2 C/\Delta z^2)/p > f[z+1]/z \quad \square$$

Proof claim 3.6 a: $d(\Delta V_i/\Delta s_i)/dk < 0$ (see figure B.1) $\Rightarrow$ if k decreases then the value of s for which the first FOC is satisfied increases for each value of z $\Rightarrow$ the s(z) curve in figure 3.2 moves to the right. Moreover, $\lim_{k \downarrow 0} s(z) = \infty$. Hence, for some $r > 0$ the s(z) and z(s) curves must intersect for $k < r$. □

b: $d(\Delta V_i/\Delta z_i)/d(\Delta C/\Delta z) < 0$ (see figure B.2) $\Rightarrow$ if $\Delta C/\Delta z$ decreases then the value of z for which the second FOC is satisfied increases for each value of s $\Rightarrow$ the z(s) curve in figure 3.2 moves upwards. Moreover, $\lim_{\Delta C/\Delta z \downarrow 0} z(s) = \infty$. Hence, for some $r > 0$ the s(z) and z(s) curves must intersect when $\Delta C(z)/\Delta z < r$ for all z. □

Proof claim 3.7 a: $d(\Delta V_i/\Delta s_i)/dp = d(p \cdot Pr \cdot F[z-1])/dp$

$$= Pr \cdot F[z-1] + p \cdot dPr/dn \cdot dn/dp \cdot F[z-1] + p \cdot Pr \cdot dF[z-1)/d\mu \cdot d\mu/dPr \cdot dPr/dn \cdot dn/dp$$

$$= Pr \cdot F[z-1] + p \cdot Pr/n \cdot dn/dp \cdot \{F[z-1] - \mu \cdot f[z-1)\}$$

$\Rightarrow d(\Delta V_i/\Delta s_i)/dp > 0$ if $dn/dp \cdot p/n \cdot \{F[z-1] - \mu \cdot f[z-1)\} > -F[z-1]$

$\Rightarrow$ if $\{F[z-1] - \mu \cdot f[z-1)\} > 0$ this yields the condition

$\varepsilon < F[z-1]/\{F[z-1] - \mu \cdot f[z-1)\}$, while

if $\{F[z-1] - \mu \cdot f[z-1)\} < 0$ we get the condition

$\varepsilon > F[z-1]/\{F[z-1] - \mu \cdot f[z-1)\}$, which is satisfied for any $\varepsilon > 0$.

Hence the s(z) curve may move leftwards or rightwards depending upon these conditions, which not only concern the price-elasticity of the demand but also z and μ. □

b: $d(\Delta V_i/\Delta z_i)/dp = d\{p \cdot (1 - F[z])\}/dp$

$$= 1 - F[z] + p \cdot -dF[z]/d\mu \cdot d\mu/dn \cdot dn/dp$$

$$= 1 - F[z] + p \cdot f[z] \cdot \mu/n \cdot dn/dp$$

$\Rightarrow d(\Delta V_i/\Delta z_i)/dp > 0$ if $dn/dp \cdot p/n > (-1 + F[z])/(\mu \cdot f[z])$

$\Rightarrow$ if $\varepsilon < (1 - F[z])/(\mu \cdot f[z])$ then $d(\Delta V_i/\Delta z_i)/dp > 0$

Hence the z(s) curve shifts upwards if this condition concerning the demand side is satisfied. □

Proof claim 3.8 a: $d(\Delta V_i/\Delta s_i)/d(n/m)$

$$= p \cdot \{dPr/d(n/m) \cdot F[z-1] + Pr \cdot dF[z-1]/d\mu \cdot d\mu/dPr \cdot dPr/d(n/m)\}$$

$$= p \cdot dPr/d(n/m) \cdot \{F[z-1] - \mu \cdot f[z-1]\}$$

The term between brackets may be positive or negative depending upon z and μ, while the rest is positive. Hence the s(z) curve may move leftwards or rightwards depending upon z and μ. □

b: $d(\Delta V_i/\Delta z_i)/d(n/m) = -p \cdot dF[z]/d\mu \cdot d\mu/d(n/m)$

$$= p \cdot f[z] \cdot d\mu/d(n/m) > 0$$

$\Rightarrow$ the z(s) curve moves upwards towards the 45° line. □

Proof claim 3.9: $\lim_{n/m\to\infty} Pr(s) = \lim_{n/m\to\infty} n/(m \cdot s) \cdot (1\text{-}e^{-m \cdot s/N}) = 1$

That is, each signal sent will surely lead to a consumer visiting its sender $\Rightarrow q = s$. We know $x = \min\{q, z\} \Rightarrow x = \min\{s, z\}$. Cost minimization requires $s = z$. Hence the firm's payoff $V = p \cdot z - C(z) - k \cdot z \Rightarrow$ FOC: $\Delta V/\Delta z{=}0 \Rightarrow z(s) = \{z\text{: } p - k = \Delta C(z)/\Delta z\}$ □

Proof claim 3.10: According to the Envelop Theorem we only have to consider the direct effects of changes in the parameters (because $\Delta V_i/\Delta s_i = \Delta V_i/\Delta z_i = 0$).

a: $dV_i/dk = -dK(s)/dk = -s < 0$ □

b: $dV_i/d(\Delta C/\Delta z) = -dC(z)/d(\Delta C/\Delta z) = -z < 0$ □

c: Rewrite $V = p \cdot \{\mu \cdot F[z-1] + z \cdot (1 - F[z])\} - C(z) - K(s)$ (see claim 3.3).

$dV_i/dp = \mu \cdot F[z-1] + z \cdot (1 - F[z]) + dV_i/d\mu \cdot d\mu/dn \cdot dn/dp$

$\quad = \mu \cdot F[z-1] + z \cdot (1 - F[z]) + p \cdot F[z-1] \cdot \mu/n \cdot dn/dp$

$\Rightarrow dV_i/dp > 0$ if $dn/dp \cdot p/n > \{-\mu \cdot F[z-1] - z \cdot (1 - F[z])\}/(\mu \cdot F[z-1])$

$\Rightarrow dV_i/dp > 0$ if $\varepsilon < 1 + \{z \cdot (1 - F[z])\}/(\mu \cdot F[z-1])$ □

d: $dV_i/d(n/m) = p \cdot \{F[z-1] + \mu \cdot dF[z-1]/d\mu - z \cdot dF[z]/d\mu\} \cdot d\mu/d(n/m)$

$= p \cdot F[z-1] \cdot d\mu/d(n/m)$ (see claim 3.3).

$d\mu/d(n/m) > 0$ and hence $dV_i/d(n/m) > 0$ □

Appendix C: Characterization of a SNE

Proof proposition 3.4: The numbers of agents in the economy influence the economic environment through the stochastic distribution of signals and the stochastic demand directed to any firm. The first can be characterized by the parameter of a Poisson distribution λ, with $\lambda =$ S/N is the expected number of signals received by any given agent. In case of symmetry we get $\lambda = (m/N) \cdot s$.
The stochastic demand directed to any firm is characterized by a Poisson distribution with parameter $\mu = (s/S) \cdot n \cdot (1 - e^{-S/N})$, which gives in case of symmetry
$\mu = (n/m) \cdot (1 - e^{-(m/N) \cdot s})$.
Substitute $m = \alpha \cdot m$, $n = \alpha \cdot n$ and $N = \alpha \cdot N$, and observe that both λ and μ remain the same. Hence, for both consumers and firms nothing changes. □

Proof proposition 3.6: Firm i's effective supply z^* is determined by the intersection of the s(z) and the z(s) curve. These two curves are defined by equation [3.2]. As we see in claim 3.3, one of the arguments of these equations is F[z]. This is the cumulative Poisson distribution function of the stochastic demand directed to firm i. Its expected value is μ, and one of the parameters of μ is the number of interested consumers, given the price, n(p). □

Proof proposition 3.7: The case of a firm i is already considered in the text: Prob $[(x_i - z_i) > 0] > 0$. Here we derive the probability that an interested consumer j (i.e. $z_j > 0$) is rationed. In a SNE the probability for a buyer to be rationed because of lack of communication is $e^{-\lambda}$, where $\lambda = m{\cdot}s/N$. $0 < e^{-\lambda} < 1$ for $\lambda > 0$. If, instead, an interested consumer j receives one or more signals, he randomly chooses one firm to visit. This firm's supply is z. As customers are served on a first-come first-served basis, the probability to obtain its demand, then, is the probability to be among the first z customers in this firm's 'queue', every place being equally probable. The number of visitors for a firm is represented by a Poisson distribution with parameter μ: f[q]. When we approximate the number of 'rival' customers visiting this firm by the same Poisson distribution, the probability that a buyer j, having received at least one signal, will be in the position to buy one unit is:

$$\begin{aligned}\text{Prob [early enough]} &= F[z-1] + \textstyle\sum_{q=z}^{\infty} \{f[q] \cdot z/(q+1)\} \\ &= F[z-1] + z/\mu \cdot \textstyle\sum_{q=z+1}^{\infty} f[q] \\ &= F[z-1] + z/\mu \cdot (1 - F[z])\end{aligned}$$

Thus, the probability that any given buyer j will not succeed in finding one unit in the period under consideration and will be rationed is:

$$\text{Prob } [(x_j - z_j) < 0] = 1 - (1 - e^{-\lambda}) \cdot \{F[z-1] + z/\mu \cdot (1 - F[z])\} > 0$$

(This equation can be made more transparent:

$$\begin{aligned}\text{Prob } [(x_j - z_j) < 0] &= 1 - (1 - e^{-\lambda}) \cdot 1/\mu \cdot \{\mu \cdot F[z-1] + z \cdot (1 - F[z])\} \\ &= 1 - (1 - e^{-\lambda}) \cdot 1/\{(n/m) \cdot (1 - e^{-\lambda})\} \cdot Ex \\ &= 1 - (m \cdot Ex)/n \text{ , where } m \cdot Ex = \text{ expected aggregate sales} \\ &\qquad\qquad\qquad\qquad\quad n = \text{ aggregate demand}\end{aligned}$$

)

Drawing i from B and j from D independently, we get Prob $[(x_i - z_i) \cdot (x_j - z_j) < 0] > 0$ for each pair $i \in B$, $j \in D$ and $z_j > 0$. □

Proof proposition 3.8: Just as in the non-cooperative case, the optimal strategy is the solution of a system of two equations. But now the FOCs have to be taken not only with respect to firm i's own strategy t_i , but also with respect to the strategies of the other firms $\mathbf{t}_{-i}$, because a change in t_i implies a simultaneous, equivalent change in $\mathbf{t}_{-i}$. Thus, a SCE is a solution to the following system of two equations:

$$\Delta V_i/\Delta s_i + \Delta V_i/\Delta \mathbf{s}_{-i} = 0$$

$$\Delta V_i/\Delta z_i + \Delta V_i/\Delta \mathbf{z}_{-i} = 0$$

From equation [3.2] we see that $\mathbf{z}_{-i}$ doesn't enter firm i's payoff, i.e. $\Delta V_i/\Delta \mathbf{z}_{-i} = 0$. Hence, the second of the FOCs doesn't change and the z(s) curve in figure 3.2 remains the same.

Turning to the FOC with respect to signaling, we see that

$$\Delta R_i/\Delta \mathbf{s}_{-i} = p \cdot \Delta\mu_i/\Delta \mathbf{s}_{-i} \cdot F[z-1] - z \cdot \Delta F[z]/\Delta\mu_i \cdot \Delta\mu_i/\Delta \mathbf{s}_{-i}$$

$$= p \cdot \Delta\mu_i/\Delta \mathbf{s}_{-i} \cdot \{F[z-1] + z \cdot f[z]\} < 0$$ because $\Delta\mu_i/\Delta \mathbf{s}_{-i} < 0$ (see claim 3.1).

That is, an increase in the signaling activity by each of the other firms ($\mathbf{s}_{-i}$) implies a decrease in the expected number of visitors for firm i $\mu(s_i, S_{-i})$. Hence, the revenue for firm i of sending one additional signal will be lower when also all other firms simultaneously send one additional signal, than in the case where firm i had to take the strategies of the other firms as given. Thus, the $\Delta R_i/\Delta s$ curves will be below the $\Delta R_i/\Delta s_i$ curves in figure B.1, and for every value of z the value of s for which this FOC is satisfied will be lower, i.e. the new s(z) curve will be at the left of the s(z) curve in figure 3.2. As a result the intersection of the s(z) and z(s) curves will occur at values z^c and s^c which are lower than z^* and s^*.

That $V^c > V^*$ follows from definition 3.3 and the fact that $t^c \neq t^*$. □

Proof proposition 3.9: Prob [cons. rationed] = 1 − (m · Ex)/n, where

$Ex = \mu \cdot F[z-1] + z \cdot (1 - F[z])$

$$\Delta Ex/\Delta s = \Delta\mu/\Delta s \cdot F[z-1] + \mu \cdot \Delta F[z-1]/\Delta\mu \cdot \Delta\mu/\Delta s - z \cdot \Delta F[z]/\Delta\mu \cdot \Delta\mu/\Delta s$$
$$= \Delta\mu/\Delta s \cdot F[z-1]$$

$\Delta\mu/\Delta s > 0$ (see claim 3.2) $\Rightarrow \Delta Ex/\Delta s > 0 \Rightarrow$ ΔProb [cons. rationed]/Δs < 0 ⇒ as s decreases the Prob [cons. rationed] increases.

Next, we consider the effect of the change in z. $Ex = z - \sum_{q=0}^{z-1} F[q]$ (see claim 3.3).
$\Rightarrow \Delta Ex/\Delta z = 1 - F[z] > 0 \Rightarrow$ as z decreases the Prob [cons. rationed] increases.
□

Proof claim of note 18: Suppose agent j has sent signals to or received signals from r agents {1, .., r} and define their aggregate effective demand resp. supply: $Z_r^+ = \sum_{i=1}^r \max(z_i, 0)$ resp. $Z_r^- = \sum_{i=1}^r \min(z_i, 0)$, and their aggregate actual purchases resp. sales: $X_r^+ = \sum_{i=1}^r \max(x_i, 0)$ resp. $X_r^- = \sum_{i=1}^r \min(x_i, 0)$. Then $(z_i - x_i) > 0$ implies $(Z_r^- - X_r^-) = 0$ and $(z_i - x_i) < 0$ implies $(Z_r^+ - X_r^+) = 0$

In words, if agent j is rationed in his demand (supply) then among the agents with whom he is in the market, i.e. among the agents with whom he has direct contact, there will be no agent with an unsatisfied supply (demand) for that commodity. If agent j is a consumer and his shopping activity is only bounded by his own desire for the commodity and his knowledge of the firms, he will continue to search for a unit of the commodity either until he has found it or until he is sure that no firm known to him is able to sell him one unit. Hence, if agent j is a consumer and has some units left at the end of the period, necessarily all consumers who know about him must have fulfilled their demand.

In a more general setting with more commodities, apparently a problem might arise if an agent would not be able to fulfil his demand not because of lack of trading partners, but because of lack of liquidity. Two assumptions serve to rule out this possibility. The first is that each agent formulates his effective demands at the beginning of each period subject to the restriction to meet his budget constraint with probability 1. This is a standard assumption in stochastic rationing models (see e.g. Green [1980]). The second assumption concerns the status of the effective demands which each agent has to take into account when deciding at the beginning of the period. We assume that if an agent meets trading partners and hasn't yet realized his whole effective demand, he will fulfil his effective demand as far as possible given the demand of his trading partners. Such an assumption, which in fact says that agents do not make new decisions during a basic period, is standard in a conceptually properly defined period model.

Appendix D: Numerical Analysis

We analyze numerically for which parameter values the FOC$^+$s and SOC for maximization of firm i's payoff V_i are fulfilled, with $V_i > 0$, when firm i chooses a strategy t^* given that all other firms choose the same strategy t^*. The parameters are k, p, N, m and those concerning the functions n(.) and C(.).

For matters of convenience of the presentation, in the numerical analysis we restrict the production cost function C(.) to be such that $\Delta C(z)/\Delta z$ is linear through the origin, and hence $\Delta^2C(z)/\Delta z^2 = c$, with $c > 0$ (see corollary 3.1). We normalize p=1 and fix N = 100,000. So, the parameters to consider are k, c, n and m.

In figure 3.3.a we consider the importance of the numbers of firms (m) and interested consumers (n), fixing k and c. It seems reasonable to assume that the marginal costs of signaling k are a relatively small fraction of the price p. For example, sending a letter won't cost much more than a stamp, and other means of signaling might be even cheaper. We fix k = 0.01 and c = 0.001. As the marginal cost of production is c·z, the value of c chosen implies that a firm will never produce more than 1,000 units.

In figure 3.3.b the role of the values of the cost parameters k and c is considered, fixing the parameters m = 100 and n = 10,000. We put m relatively small to n in order to mirror the 'division of labor'.

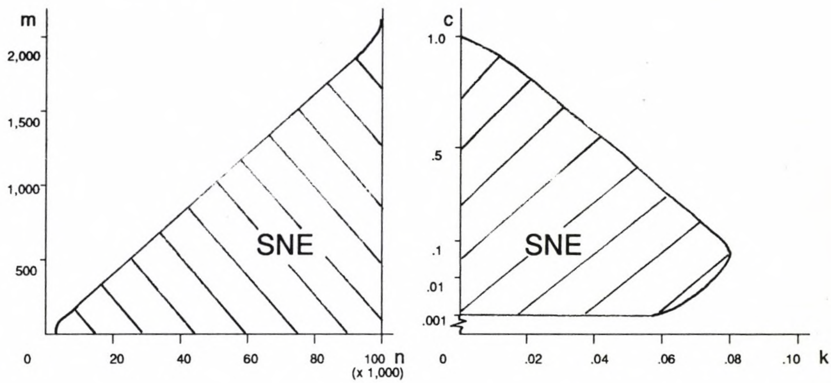

fig. 3.3 existence of SNE

In figure 3.3.a the linearity of the boundary is prominent. Except for very small or very large values of n, a SNE exists, given the other parameters, as long as n/m > 50. That is, on average there should be at least 50 interested consumers per firm in the population, which does not necessarily mean that firms should actually get this number of clients.

In figure 3.3.b we used a somewhat unconventional scale on the vertical axis for expositional reasons. Clearly, k should be smaller than .10 for a SNE to exist as $(n/N)\cdot p = .10$ (see proposition 3.2). Essentially there are two reasons why a SNE does not exist for some parameter values. For combinations of c and k below the shaded area a SNE doesn't exist because the SOC is not fulfilled (remember $c = \Delta^2 C/\Delta z^2$). For values above the shaded area the FOC or the condition V>0 are not satisfied.

Appendix E: Notation

A	set of all agents
B	set of firms
C(z)	production function
D	set of consumers
$\Delta h(x)$	$h(x+\Delta x) - h(x)$
ε	price-elasticity of demand
$f[q\mid \mu]$	p.d.f. of q with parameter μ
F[z]	$\sum_{q=0}^{z} f[q]$
k	'marginal' cost of signaling
K(s)	signaling function
λ	expected number of signals received by any given agent
m	number of firms
μ_i	expected demand directed to firm i
n	number of interested consumers

N	number of agents in the economy
ω_i	endowments agent i
p	price of the commodity
$\overline{p}_i$	threshold price of consumer i
Pr	probability that any given signal leads to a consumer visiting its sender
q_i	demand directed towards firm i
$R_i(.)$	gross revenue firm i
s_i	number of signals sent by firm i
S_{-i}	aggregate number of signals sent by all other firms
S	aggregate number of signals sent by all firms
t^*	SNE strategy t
t^c	SCE strategy t
$\hat{t}$	strategy t for which the FOC-plus-symmetry-conditions are satisfied
$t_i \equiv (z_i, s_i)$	strategy of firm i
$\mathbf{t} \equiv (\mathbf{z}, \mathbf{s})$	complete vector of strategies of all firms
$\mathbf{t}_{-i} \equiv (\mathbf{z}_{-i}, \mathbf{s}_{-i})$	vector of strategies of all other firms
τ	time-index
$U_i(.)$	utility function consumer i
$V_i(.)$	payoff to firm i
x_i	actual transactions by firm i
$X^{+(-)}$	aggregate actual purchases (sales)
z_i	output or effective supply of firm i
$Z^{+(-)}$	aggregate demand (supply)

Notes

1. For the literature about the informational efficiency of allocation mechanisms we refer to Calsamiglia [1987]).

2. Besides this the auctioneer must, of course, check that the rules of the game are respected. Thus e.g. trades must be such that each individual's budget constraint is obeyed.

3. To give one example of the costs (disutility) of uninformed search: A smoker, after

having troubled two non-smokers vainly for a light, might hesitate before troubling the next passer-by, and will probably wait until he perceives someone smoking.

4. Cf. Gilles & Ruys [1988] who take the exactly opposite approach, assuming a situation of complete information of each agent with respect to the deterministic relational structure of the economy.

5. We differ e.g. from the so-called 't-wise optimality' literature. Goldman & Starr (1982), generalizing results of Rader (1968) and Feldman (1973), show that if there are some traders who deal in all commodities at least in small quantities or if there is one good which everyone values and possesses in positive amounts, t-wise optimality implies Pareto optimality. This is a property of the final allocation and does not go into details concerning the trade process by which it may be achieved. Feldman (1973) did give such a possible process: an infinitely rotating sequence of bilateral trades. Every agent is supposed to act directly (although sequentially) with all other agents, and the accounts are made up only after an infinite sequence. In fact, Feldman assumes that transaction costs are zero in case of bilateral trade. While it seems reasonable to assume that trade between only two agents does not involve transaction costs, it seems much less plausible that the formation of an infinite number of trading pairs itself is a costless affair.

6. Note that the signals give no information about the size of the effective demands, and that this corresponds to what we usually observe in reality. A reason might be the following. Stating an effective demand creates in a certain sense a commitment, but if the nth agent arrives and reminds agent i of such a commitment agent i might assert that he has fulfilled already part of his commitment. As this is difficult to check and may lead to confusion, agent i will prefer to state only the size of the remaining demand directly to the nth agent when he arrives. Note also that in discrete-time models agents make their effective demand decisions only at the beginning of each period (see section 3.3), i.e. they don't deviate from these by making new decisions during the period, even when they have not publicly announced their effective demand decisions.

7. To assume that consumers may make only one visit and that the order in which they make their visit is random is convenient for presentational reasons. it does not restrict the nature of the problem of the firm in any sense; it only changes the value of some of its parameters. it is just a simplified version of the following scenario which would apply when one would consider more commodities and more visits.
Assume that the economic environment of each agent is such that his transaction attempts cannot take place continuously but only at discrete times, and that individual actions are discrete but not synchronized among agents. A method of modeling transaction attempts would be to let time flow continuously and to view the visits and exchanges as discrete events of zero duration like the arrivals in a Poisson process (see Foley [1975] or Diamond [1982]). An operational counterpart to solve problems like order of visits and simultaneity may then be found in the theory of interacting particle systems (see e.g. Griffeath [1979]).

Associating with each agent a 'random clock' which rings, independently for each agent, at the instances of a Poisson process, an agent may do a transaction attempt when his clock rings. As long as the length of a period τ isn't infinite compared with the parameter of the Poisson process of the 'random clock', each consumer will be able to make only a limited number of visits in each period.

8. In this sense we differ from Ioannides [1990] who also analyses communication by individual agents in order to make markets, and then considers multilateral trade between all agents who are, directly or indirectly, informationally linked.

9. What we mean by 'randomly' in a strictly technical sense is explained in appendix A. Conceptually, we don't want to exclude the possibility that agents make their choices in a deterministic manner. However, as they do not have any relevant information on which to base their choice, and as we don't have any insights which irrelevant criteria they might apply, we may very well consider their choices to be random.

10. Thus, in our model there is no need to distinguish the threshold price (the price above which a consumer does not buy) from the so-called reservation price as it is known from the search-literature. The latter is the price below which you do immediately buy and stop searching. This is not a purely individual characteristic, but depends upon the market situation. More specifically, it will depend upon the assumed trading structure of the economy and upon the strategies chosen by the other agents.

11. See note 9.

12. To lighten notational burden somewhat, we will usually write n instead of n(p).

13. Allowing for more visits per buyer would just give a higher value for the parameter μ.

14. In this sense we differ e.g. from the literature on stochastic rationing (e.g. Green [1980]) where it is directly assumed that each agent's trading possibilities are a stochastic function only of his own demand and the aggregate demand and supply in the economy. We also differ from the literature on completely random matching models (e.g. Gale [1985]) where an agent's trading opportunities are independent from his own decisions. and we differ from the fix-price literature in general where the sending of effective demands is the only means of communication (cf. Drazen's [1980] criticism).

15. For reasons of expositional convenience we will obscure somewhat the fact that the variables z and s are discrete. Hence, considering unit increments of these variables, the FOCs are:

$\Delta V_i(z_i, s_i, S_{-i})/\Delta s_i > 0$ while $\Delta V_i(z_i, s_i+\Delta s, S_{-i})/\Delta s_i \leq 0$

$\Delta V_i(z_i, s_i, S_{-i})/\Delta z_i > 0$ while $\Delta V_i(z_i+\Delta z, s_i, S_{-i})/\Delta z_i \leq 0$

This is also important with respect to the existence of a SNE, which usually doesn't exist in cases of pure strategies in a discrete choice problem. In fact, we suppose that these variables will assume large values such that they are approximately continuous.

16. Anticipating a result of the analysis, we avoid here to give a rather cumbersome analogous expression for the case in which $\Delta^2 C(z)/\Delta z^2 < 0$.

17. An additional condition is that the parameters remain such as to allow for the Poisson approximations.

18. Allowing for more visits per buyer would not change the picture. Each buyer unsatisfied in his first round might be more successful in his second or third round. As a result, given the level of signaling s, the probability of rationing will be lower for both firms and buyers. But only if consumers could visit all firms they know, individual markets would be orderly (a proof of this claim can be found in appendix C). Note, however, that still, at the aggregate level the economy would not be orderly, and both firms and consumers might rationed at the same time.

19. In this respect the model differs from some other models with stochastic rationing (e.g. Weinrich [1984]), where it is usually assumed that markets are orderly, implicitly assuming some kind of 'central lottery'.

References

Benassy, J.-P. (1982), *The Economics of Market Disequilibrium*, New York, Academic Press

Blin, J.-M. (1980), Comments on the Economics of Markets: A Simple Model of the Market-making Process, *Journal of Business*, Vol.53, no.3, pt.2, p. S193-S197

Calsamiglia, X. (1987), Informational Requirements of Parametric Resource Allocation Processes, in: T. Groves, R. Radner and S. Reiter (eds.) (1987), *Information, Incentives, and Economic Mechanisms*, Univ. of Minnesota Press

DeGroot, M.H. (1986), Probability and Statistics, 2nd ed., Addison-Wesley, Reading

Dehez, P. (1980), Employment and Dividend Policy of the Firm under Risk, *Review of Economic Studies*, 47, p. 503-511

Diamond, P.A. (1982), Aggregate Demand Management in Search Equilibrium, *Journal of Political Economy*, 51, p. 881-894

Drazen, A. (1980), Recent Developments in Macroeconomic Disequilibrium Theory, *Econometrica*, Vol.48, March, no.2

Feldman, A. (1973), Bilateral Trading Processes, Pairwise Optimality, and Pareto Optimality, *Review of Economic Studies*, 40, p. 463-473

Fisher, F.M. (1983), *Disequilibrium Foundations of Equilibrium Economics*, Cambridge University Press

Fisher, F.M. (1989), It Takes t^* to Tango: Trading Coalitions with Fixed Prices, *Review of Economic Studies*, 56, p. 391-404

Foley, D.K. (1975), On Two Specifications of Asset equilibrium in Macroeconomic Models, *Journal of Political Economy*, Vol.83, no.2, p. 303- 324

Foley, D.K. & M. Hellwig (1975), Asset Management with Trading Uncertainty, *Review of Economic Studies*, 42, p. 327-346

Gale, D. (1985), Limit Theorems for Markets with Sequential Bargaining, *CARESS Working Paper* 85-15, Univ. of Pennsylvania

Gary-Bobo, R. & J.-P. Lesne (1988), Advertising Strategies and Competition in a Chamberlinian Market Model, research paper Cath. Univ. de Louvain

Gilles, R. & P. Ruys (1988), Characterization of Economic Agents in Arbitrary Communication Structures, *Research Memorandum FEW* 365, Tilburg Univ.

Goldman, S. & R. Starr (1982), Pairwise, t-wise, and Pareto Optimalities, *Econometrica*, Vol.50, no.3, May, p. 593-606

Gould, J. (1980), The Economics of Markets: A Simple Model of the Market- making process, *Journal of Business*, Vol.53, no.3, pt.2, p. S167-S187

Grandmont, J.M. (1988), Temporary Equilibrium Theory: An Addendum, in: Grandmont, J.M. (ed.) (1988), *Temporary Equilibrium, Selected Readings*, New York

Green, J.R. (1980), On the Theory of Effective Demand, *Economic Journal*, 90 (358), June, p. 341-353

Griffeath, D. (1979), *Additive and Cancellative Interacting Particle Systems*, Lecture Notes in Mathematics 724, Springer-Verlag, Berlin

Hahn, F.H. (1980), Discussion, in: Kareken, J. & N. Wallace (eds.), *Models of Monetary Economics*, Fed. Res. Bank of Minneapolis

Hahn, F.H. (1988), Liquidity, *Discussion Paper* No.129, Univ. of Cambridge

Ioannides, Y.M. (1990), Trading Uncertainty and Market Form, *International Economic Review*, Vol.31, No.3, August.

Kirman, A. (1983), Communication in Markets, A Suggested Approach, *Economics Letters*, 12, p. 101-108

Kormendi, R.C. (1979), Dispersed Transaction Prices in a Model of Decentralized Pure Exchange, in: S.A. Lippman & J.J. McCall (eds.), *Studies in the Economics of Search*, North Holland, Amsterdam

Rader, T. (1968), Pairwise Optimality and Non-Competitive Behavior, in: J.P. Quirk & A.M. Zarley (eds.), *Papers in Quantitative Economics*, Univ. Press of Kansas, Lawrence, Kansas

Shubik, M. (1975), The General Equilibrium Model Is Incomplete and Not Adequate for the Reconciliation of Micro and Macroeconomic Theory, *Kyklos*, 28 (3), p. 545-573

Smith, A. (1776), *An Enquiry into the Nature and Causes of the Wealth of Nations*, London

Spence, A.M. (1974), *Market Signalling*, Harvard Univ. Press

Ulph, A. & D. Ulph (1975), Transaction Costs in General Equilibrium Theory - A Survey, *Economica*, November, p. 335-372

Varian, H.R. (1984), *Microeconomic Analysis*, 2nd ed., Norton, New York

Weinrich, G. (1984), On the Theory of Effective Demand under Stochastic Rationing, *Journal of Economic Theory*, Vol.34, No.1, October, p. 95-115

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